Reprint ISSN 0973-9424

INTERNATIONAL JOURNAL OF MATHEMATICAL SCIENCES AND ENGINEERING APPLICATIONS

(IJMSEA)

ASCENT PUBLICATION @ ASCENT PUNE, INDIA

www.ascent-journals.com

International J. of Math. Sci. & Engg. Appls. (IJMSEA)
ISSN 0973-9424, Vol. 17 No. II (December, 2023), pp. 17-23

COMMON MINIMAL EQUITABLE DOMIATING SYMMETRIC n-SIGRAPHS

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Abstract

In this paper, we define the common minimal equitable dominating symmetric n-sigraph of a given symmetric n-sigraph and offer a structural characterization of common minimal equitable dominating symmetric n-sigraphs. In the sequel, we also obtained switching equivalence characterization $\overline{S_n} \sim CMED(S_n)$ where $\overline{S_n}$ and $CMED(S_n)$ are complementary symmetric n-sigraph and common minimal equitable dominating symmetric n-sigraph of a symmetric n-sigraph S_n respectively.

1. Introduction

Unless mentioned or defined otherwise, for all terminology and notion in graph theory the reader is refer to [2]. We consider only finite, simple graphs free from self-loops.

Let $n \geq 1$ be an integer. An *n-tuple* $(a_1, a_2, ..., a_n)$ is *symmetric*, if $a_k = a_{n-k+1}, 1 \leq k \leq n$. Let $H_n = \{(a_1, a_2, ..., a_n) : a_k \in \{+, -\}, a_k = a_{n-k+1}, 1 \leq k \leq n\}$ be the set of

Key Words and Phrases : *Symmetric n-sigraph, Symmetric n-marked graph, Balance, Switching, Common minimal equitable dominating symmetric n-sigraphs, Complementation.*

2000 AMS Subject Classification : 05C22.

all symmetric n-tuples. Note that H_n is a group under coordinate wise multiplication, and the order of H_n is 2^m, where $m = \lceil \frac{n}{2} \rceil$.

A *symmetric n-sigraph (symmetric n-marked graph)* is an ordered pair $S_n = (G, \sigma)$ ($S_n = (G, \mu)$), where $G = (V, E)$ is a graph called the *underlying graph* of S_n and $\sigma : E \to H_n$ ($\mu : V \to H_n$) is a function.

In this paper by an *n-tuple/n-sigraph/n-marked graph* we always mean a symmetric n-tuple/symmetric n-sigraph/symmetric n-marked graph.

An n-tuple $(a_1, a_2, ..., a_n)$ is the *identity n-tuple*, if $a_k = +$, for $1 \leq k \leq n$, otherwise it is a *non-identity n-tuple*. In an n-sigraph $S_n = (G, \sigma)$ an edge labelled with the identity n-tuple is called an *identity edge*, otherwise it is a *non-identity edge*.

Further, in an n-sigraph $S_n = (G, \sigma)$, for any $A \subseteq E(G)$ the *n-tuple* $\sigma(A)$ is the product of the n-tuples on the edges of A.

In [8], the authors defined two notions of balance in n-sigraph $S_n = (G, \sigma)$ as follows (See also R. Rangarajan and P.S.K.Reddy [4]).

Definition : Let $S_n = (G, \sigma)$ be an n-sigraph. Then,

(i) S_n is *identity balanced* (or *i-balanced*), if product of n-tuples on each cycle of S_n is the identity n-tuple, and

(ii) S_n is *balanced*, if every cycle in S_n contains an even number of non-identity edges.

Note: An i-balanced n-sigraph need not be balanced and conversely.

The following characterization of i-balanced n-sigraphs is obtained in [8].

Theorem 1.1 ***(E. Sampathkumar et al. [8])*** : An n-sigraph $S_n = (G, \sigma)$ is i-balanced if, and only if, it is possible to assign n-tuples to its vertices such that the n-tuple of each edge uv is equal to the product of the n-tuples of u and v.

In [8], the authors also have defined switching and cycle isomorphism of an n-sigraph $S_n = (G, \sigma)$ as follows: (See also [3], [5-7], [10-15], [17-21]).

Let $S_n = (G, \sigma)$ and $S'_n = (G', \sigma')$, be two n-sigraphs. Then S_n and S'_n are said to be *isomorphic*, if there exists an isomorphism $\phi : G \to G'$ such that if uv is an edge in S_n with label $(a_1, a_2, ..., a_n)$ then $\phi(u)\phi(v)$ is an edge in S'_n with label $(a_1, a_2, ..., a_n)$.

Given an n-marking μ of an n-sigraph $S_n = (G, \sigma)$, *switching* S_n with respect to μ is the operation of changing the n-tuple of every edge uv of S_n by $\mu(u)\sigma(uv)\mu(v)$. The n-

sigraph obtained in this way is denoted by $\mathcal{S}_\mu(S_n)$ and is called the *μ-switched n-sigraph* or just *switched n-sigraph*.

Further, an n-sigraph S_n *switches* to n-sigraph S'_n (or that they are *switching equivalent* to each other), written as $S_n \sim S'_n$, whenever there exists an n-marking of S_n such that $\mathcal{S}_\mu(S_n) \cong S'_n$.

Two n-sigraphs $S_n = (G, \sigma)$ and $S'_n = (G', \sigma')$ are said to be *cycle isomorphic*, if there exists an isomorphism $\phi : G \to G'$ such that the n-tuple $\sigma(C)$ of every cycle C in S_n equals to the n-tuple $\sigma(\phi(C))$ in S'_n.

We make use of the following known result (see [8]).

Theorem 1.2 ***(E. Sampathkumar et al. [8])*** : Given a graph G, any two n-sigraphs with G as underlying graph are switching equivalent if, and only if, they are cycle isomorphic.

Let $S_n = (G, \sigma)$ be an n-sigraph. Consider the n-marking μ on vertices of S defined as follows: each vertex $v \in V$, $\mu(v)$ is the product of the n-tuples on the edges incident at v. *Complement* of S is an n-sigraph $\overline{S_n} = (\overline{G}, \sigma')$, where for any edge $e = uv \in \overline{G}$, $\sigma'(uv) = \mu(u)\mu(v)$. Clearly, $\overline{S_n}$ as defined here is an i-balanced n-sigraph due to Theorem 1.1.

2. Common Minimal Equitable Dominating n-Sigraph of an n-Sigraph

A subset D of $V(\Gamma)$ is called an *equitable dominating set* of a graph Γ, if for every $v \in V - D$ there exists a vertex $v \in D$ such that $uv \in E(\Gamma)$ and $|d(u) - d(v)| \leq 1$. The minimum cardinality of such a dominating set is denoted by γ_e and is called equitable domination number of Γ. An equitable dominating set D is *minimal*, if for any vertex $u \in D$, $D - \{u\}$ is not a equitable dominating set of Γ. This concept was introduced by Deepak et al. [1].

In [1], the authors introduced a new class of intersection graphs in the field of domination theory. The common minimal equitable dominating graph is denoted by $CMED(\Gamma)$ is the graph which has the same vertex set as Γ with two vertices are adjacent if and only if there exist minimal equitable dominating in Γ containing them.

In this paper, we introduce a natural extension of the notion of common minimal equitable dominating graph to the realm of n-sigraphs since this appears to have interesting connections with complementary n-sigraph.

The *common minimal equitable dominating n-sigraph* $CMED(S_n)$ of an n-sigraph $S_n = (G, \sigma)$ is an n-sigraph whose underlying graph is $CMED(G)$ and the n-tuple of any edge uv in $CMED(S_n)$ is $\mu(u)\mu(v)$, where μ is the canonical n-marking of S_n. Further, an n-sigraph $S_n = (G, \sigma)$ is called common minimal equitable dominating n-sigraph, if $S_n \cong CMED(S'_n)$ for some n-sigraph S'_n. In this paper we will give a structural characterization of n-sigraphs which are common minimal equitable dominating n-sigraphs.

The following result indicates the limitations of the notion $CMED(S_n)$ introduced above, since the entire class of i-unbalanced n-sigraphs is forbidden to be common minimal equitable dominating n-sigraphs.

Theorem 2.1 : For any n-sigraph $S_n = (G, \sigma)$, its common minimal equitable dominating n-sigraph $CMED(S_n)$ is i-balanced.

Proof : Since the n-tuple of any edge uv in $CMED(S_n)$ is $\mu(u)\mu(v)$, where μ is the canonical n-marking of S_n, by Theorem 1.1, $CMED(S_n)$ is i-balanced. □

For any positive integer k, the k^{th} iterated common minimal equitable dominating n-sigraph $CMED(S_n)$ of S_n is defined as follows:

$$(CMED)^0(S_n) = S_n,\ (CMED)^k(S_n) = CMED((CMED)^{k-1}(S_n)).$$

Corollary 2.2 : For any n-sigraph $S_n = (G, \sigma)$ and any positive integer k, $(CMED)^k(S_n)$ is i-balanced.

The following result characterize n-sigraphs which are common minimal equitable dominating n-sigraphs.

Theorem 2.2 : An n-sigraph $S_n = (G, \sigma)$ is a common minimal equitable dominating n-sigraph if, and only if, S_n is i-balanced n-sigraph and its underlying graph G is a common minimal equitable dominating graph.

Proof : Suppose that S_n is i-balanced and G is a $CMED(G)$. Then there exists a graph H such that $CMED(H) \cong G$. Since S_n is i-balanced, by Theorem 1.1, there exists an n-marking μ of G such that each edge uv in S_n satisfies $\sigma(uv) = \mu(u)\mu(v)$. Now consider the n-sigraph $S'_n = (H, \sigma')$, where for any edge e in H, $\sigma'(e)$ is the n-marking of the corresponding vertex in G. Then clearly, $CMED(S'_n) \cong S_n$. Hence S_n is a common minimal equitable dominating n-sigraph.

Conversely, suppose that $S_n = (G, \sigma)$ is a common minimal equitable dominating n-sigraph. Then there exists an n-sigraph $S'_n = (H, \sigma')$ such that $CMED(S'_n) \cong S_n$.

Hence G is the $CMED(G)$ of H and by Theorem 2.1, S_n is i-balanced. □

In [1], the authors characterized graphs for which $CMED(G) \cong \overline{G}$.

Theorem 2.3 ***(G. Deepak et al. [1])*** : For any graph $G = (V, E)$, $CMED(G) \cong \overline{G}$ if and only if every minimal equitable dominating set of G is independent.

We now characterize n-sigraphs whose common minimal equitable dominating n-sigraphs and complementary n-sigraphs are switching equivalent.

Theorem 2.4 : For any n-sigraph $S_n = (G, \sigma)$, $\overline{S_n} \sim CMED(S_n)$ if, and only if, every minimal equitable dominating set of G is independent.

Proof : Suppose $\overline{S_n} \sim CMED(S_n)$. This implies, $\overline{S_n} \cong CMED(S_n)$ and hence by Theorem 2.4, every minimal equitable dominating set of G is independent.

Conversely, suppose that every minimal equitable dominating set of G is independent. Then $\overline{S_n} \cong CMED(S_n)$ by Proposition 2.4. Now, if S_n is an n-sigraph with every minimal equitable dominating set of underlying graph G is independent, by the definition of complementary n-sigraph and Theorem 2.1, $\overline{S_n}$ and $CMED(S_n)$ are i-balanced and hence, the result follows from Theorem 1.2. □

Theorem 2.5 : For any two n-sigraphs S_n and S'_n with the same underlying graph, their common minimal equitable dominating n-sigraphs are switching equivalent.

Proof : Suppose $Sn = (G, \sigma)$ and $S'_n = (G', \sigma')$ be two n-sigraphs with $G \cong G'$. By Theorem 2.1, $CMED(S_n)$ and $CMED(S'_n)$ are i-balanced and hence, the result follows from Theorem 1.2. □

In any $m \in H_n$, the *m-complement* of $a = (a_1, a_2, ..., a_n)$ is: $a^m = am$. For any $M \subseteq H_n$, and $m \in H_n$, the *m-complement* of M is $M^m = \{a^m : a \in M\}$.

For any $m \in H_n$, the *m-complement* of an n-sigraph $S_n = (G, \sigma)$, written (S_n^m), is the same graph but with each edge label $a = (a_1, a_2, ..., a_n)$ replaced by a^m.

For an n-sigraph $S_n = (G, \sigma)$, the $CMED(S_n)$ is i-balanced. We now examine, the condition under which m-complement of $CMED(S_n)$ is i-balanced, where for any $m \in H_n$. For an n-sigraph $S_n = (G, \sigma)$, the $CMED(S_n)$ is i-balanced. We now examine, the conditions under which m-complement of $CMED(S_n)$ is i-balanced, where for any $m \in H_n$.

Theorem 2.6 : Let $S_n = (G, \sigma)$ be an n-sigraph. Then, for any $m \in H_n$, if $CMED(G)$ is bipartite then $(CMED(S_n))^m$ is i-balanced.

Proof : Since, by Theorem 2.1, $CMED(S_n)$ is i-balanced, for each k, $1 \leq k \leq n$, the number of n-tuples on any cycle C in $CMED(S_n)$ whose k^{th} co-ordinate are $-$ is even. Also, since $CMED(G)$ is bipartite, all cycles have even length; thus, for each k, $1 \leq k \leq n$, the number of n-tuples on any cycle C in $CMED(S_n)$ whose k^{th} co-ordinate are $+$ is also even. This implies that the same thing is true in any m-complement, where for any $m, \in H_n$. Hence $(CMED(S_n))^t$ is i-balanced. □

Theorem 2.5 & 2.6 provides easy solutions to other n-sigraph switching equivalence relations, which are given in the following results.

Corollary 2.7 : For any n-sigraph $S_n = (G, \sigma)$, $\overline{(S_n)^m} \sim CMED(S_n)$ if, and only if, every minimal equitable dominating set of G is independent.

Corollary 2.8 : For any n-sigraph $S_n = (G, \sigma)$, $\overline{S_n} \sim CMED((S_n)^m)$ if, and only if, every minimal equitable dominating set of G is independent.

Corollary 2.9 : For any n-sigraph $S_n = (G, \sigma)$, $\overline{(S_n)^m} \sim CMED((S_n)^m)$ if, and only if, every minimal equitable dominating set of G is independent.

Corollary 2.10 : For any two n-sigraphs S_n and S'_n with the same underlying graph, $CMED(S_n)$ and $CMED((S'_n)^m)$ are switching equivalent.

Corollary 2.11 : For any two n-sigraphs S_n and S'_n with the same underlying graph, $CMED((S_n)^m)$ and $CMED(S'_n)$ are switching equivalent.

Corollary 2.12 : For any two n-sigraphs S_n and S'_n with the same underlying graph, $CMED((S_n)^m)$ and $CMED((S'_n)^m)$ are switching equivalent.

Corollary 2.13 : For any two n-sigraphs $S_n = (G, \sigma)$ and $S'_n = (G', \sigma')$ with the $G \cong G'$ and G, G' are bipartite, $(CMED(S_n))^m$ and $CMED(S'_n)$ are switching equivalent.

Corollary 2.14 : For any two n-sigraphs $S_n = (G, \sigma)$ and $S'_n = (G', \sigma')$ with the $G \cong G'$ and G, G' are bipartite, $CMED(S_n)$ and $(CMED(S'_n))^m$ are switching equivalent.

Corollary 2.15 : For any two n-sigraphs $S_n = (G, \sigma)$ and $S'_n = (G', \sigma')$ with the $G \cong G'$ and G, G' are bipartite, $(CMED(S_1))^m$ and $(CMED(S_2))^m$ are switching equivalent.

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